J. Fluid Mech. (1969), *vol.* 38, *part* 4, *pp.* 673–677
Printed in Great Britain

Magnetogasdynamic thrust bearing with an axial pinch

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(Received 1 April 1969)

An analysis of a hydrostatic thrust bearing with electrically conducting compressible lubricant under an axial-current-induced pinch is presented. It is shown that the load capacity of the bearing can be increased by the pinch effect and the magnitude of the pinch effect depends on the mass flow rate. It is also shown that a load proportional to the square of the axial current can be sustained even when there is no flow or external pressurization.

1. Introduction

In recent times the axial pinch effects in magnetohydrodynamics have been utilized to increase the load capacity of bearings with electrically conducting lubricants (e.g. Elco & Hughes 1962; Ramanaiah 1966*a*, *b*, 1967). It was Elco & Hughes (1962) who showed for the first time that the use of an axial-current-induced pinch increases the load capacity without affecting the frictional drag on the rotor of a thrust bearing.

In the present paper an analysis of a hydrostatic thrust bearing with electrically conducting gas lubricant under an axial pinch is presented. It will be assumed that the lubricant is isothermal.

2. Analysis

The bearing configuration is shown in figure 1. It is assumed that the bearing surfaces are ideal conductors and a potential is applied between them. Because the lubricant is conducting an axial current density j_z will exist between the stator and rotor. This axial current density gives rise to an azimuthal magnetic induction B_θ which, in turn, interacts with j_z to provide a radial body force. This force results in a pinch effect that can alter the load capacity of the bearing. As the motion of the rotor is not affected by the pinch it will not be considered here.

Making the usual assumptions of hydromagnetic lubrication like negligible inertia, $\partial p/\partial z = 0$, $h \ll a$, where p is the pressure, z is the axial distance, h is the film thickness and a is the outer radius of the bearing (Elco & Hughes 1962), the equation of motion of the lubricant takes the form

$$0 = -\frac{dp}{dr} + \mu\frac{\partial^2 u}{\partial z^2} - j_z B_\theta, \tag{2.1}$$

where r is the radial distance, μ is the viscosity and u is the velocity of the lubricant. From Maxwell's equation $\nabla \times \mathbf{B} = \mu_e \mathbf{J}$, where μ_e is the permeability of free space, it follows that

$$B_\theta = \tfrac{1}{2}\mu_e j_z r. \tag{2.2}$$

Using (2.2) and $i = \pi a^2 j_z$ (i is the axial current) in (2.1) and introducing

$$k = \mu_e i^2/2\pi^2 a^4,$$

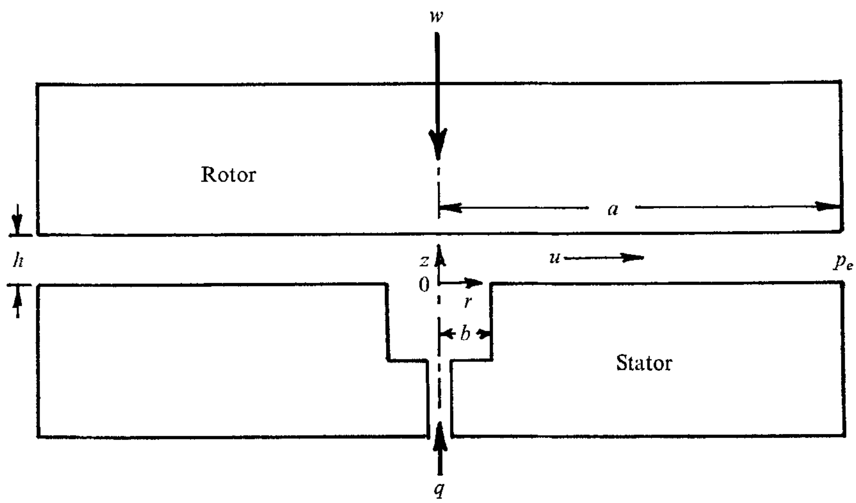

FIGURE 1. Thrust bearing.

one obtains

$$\frac{\partial^2 u}{dz^2} = \frac{1}{\mu}\left(\frac{dp}{dr} + kr\right), \tag{2.3}$$

which when solved with the boundary conditions

$$u = 0 \quad \text{at} \quad z = 0 \quad \text{and at} \quad z = h \tag{2.4}$$

yields

$$u = \frac{1}{2\mu}\left(\frac{dp}{dr} + kr\right)(z^2 - hz). \tag{2.5}$$

The mass flow rate q is given by

$$q = \int_0^k 2\pi\rho u r\, dz. \tag{2.6}$$

From (2.5) and (2.6) we obtain the equation for pressure,

$$\rho r\left(\frac{dp}{dr} + kr\right) + \frac{6\mu q}{\pi h^3} = 0, \tag{2.7}$$

where ρ is the density.

Introducing the following dimensionless quantities (where subscript e denotes external quantities)

$$\left.\begin{aligned} R &= \frac{r}{a}, \quad P = \frac{p}{p_e} = \frac{\rho}{\rho_e} \quad \text{(isothermal)}, \\ K &= \frac{ka^2}{2p_e}, \quad Q = \frac{6\mu q}{\pi h^3 p_e \rho_e}, \end{aligned}\right\} \tag{2.8}$$

equation (2.7) takes the form

$$RP\left(\frac{dP}{dR}+2KR\right)+Q=0, \tag{2.9}$$

with the condition

$$P=1 \quad \text{at} \quad R=1. \tag{2.10}$$

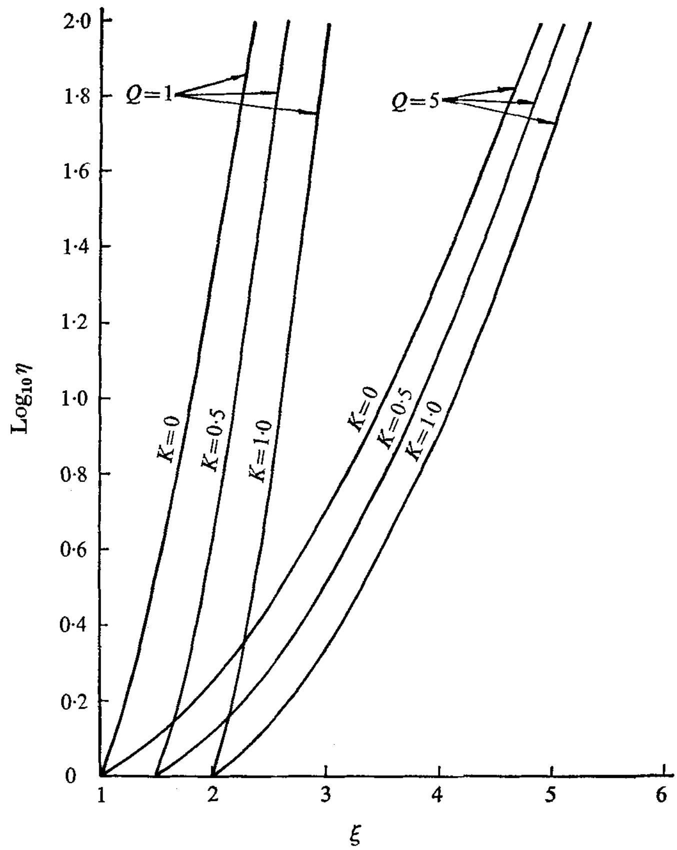

FIGURE 2. Plot of $\log_{10}\eta$ against ξ.

The non-linear equation (2.9) becomes linear,

$$\frac{d\eta}{d\xi}-\frac{2\xi\eta}{Q}=-\frac{2K}{Q}, \tag{2.11}$$

by introducing the variables

$$\xi=P+KR^2 \quad \text{and} \quad \eta=R^{-2}. \tag{2.12}$$

Solving (2.11) with the condition

$$\xi=1+K \quad \text{at} \quad \eta=1, \tag{2.13}$$

we get

$$\eta=e^{\xi^2/Q}\left[e^{-(1+K)^2/Q}-K\sqrt{\left(\frac{\pi}{Q}\right)}\left\{\operatorname{erf}\frac{\xi}{\sqrt{Q}}-\operatorname{erf}\frac{1+K}{\sqrt{Q}}\right\}\right], \tag{2.14}$$

which takes the form

$$\xi=1+K \quad \text{when} \quad Q=0 \quad \text{(no flow)} \tag{2.15}$$

and

$$\eta=e^{(\xi^2-1)/Q} \quad \text{when} \quad K=0 \quad \text{(no current)}. \tag{2.16}$$

The plot of $\log_{10}\eta$ against ξ is shown in figure 2 for $K = 0, 0{\cdot}5, 1{\cdot}0$ and $Q = 1, 5$. One can find the dimensionless pressure P for any value of R from the plot remembering (2.12). The pressure in the recess ($R = 0{\cdot}1$) is given in table 1.

The pressure obtained from (2.14) is valid only outside the recess. In the recess, since the flow rate and velocity are negligible, we obtain

$$\xi = \xi_0, \tag{2.17}$$

where ξ_0 is the value of ξ at $R = b/a$, where b is the radius of the recess.

K	Q = 0	1	5
0·0	1·00	2·37	4·90
0·5	1·50	2·65	5·11
1·0	2·00	2·98	5·34

TABLE 1. Dimensionless recess pressure P_0

K	Q = 0	1	5
0·0	0·00	0·38	1·27
0·5	0·25	0·63	1·48
1·0	0·50	1·02	1·87

TABLE 2. Dimensionless load capacity W

3. Load capacity

The load capacity of the bearing is given by

$$w = \int_0^a 2\pi r(p-p_e)\,dr = -\pi\int_0^a r^2\frac{dp}{dr}\,dr, \tag{3.1}$$

whose dimensionless form is

$$\frac{w}{\pi a^2 p_e} = W = -\int_0^1 R^2\frac{dP}{dR}dR. \tag{3.2}$$

Using the proper forms of dP/dR in the recess and outside, we obtain

$$W = \frac{K}{2} + \int_{1+K}^{\xi_0}\frac{d\xi}{\eta}. \tag{3.3}$$

The integral has been evaluated by Simpson's rule for $K = 0{\cdot}5, 1{\cdot}0$ and $Q = 1, 5$ and the results are given in table 2.

The dimensionless load W takes the form

$$W = \tfrac{1}{2}K \quad \text{when} \quad Q = 0, \tag{3.4}$$

and

$$W = \frac{\sqrt{(\pi Q)}}{2}e^{1/Q}\left[\operatorname{erf}\frac{\xi_0}{\sqrt{Q}} - \operatorname{erf}\frac{1}{\sqrt{Q}}\right] \quad \text{when} \quad K = 0. \tag{3.5}$$

The values of W for $K = 0$ and $Q = 1, 5$ are also given in table 2.

4. Conclusions

For a given mass flow rate the load capacity can be increased by axial-current-induced pinch. On the other hand, a given load can be sustained with less flow rate because of the pinch. Further, a load $\mu_e i^2/8\pi$ can be supported even when there is no flow or external pressurization.

It may be noted here that the above phenomena are the same qualitatively as that of incompressible lubricants. However, the main difference is that, while the effect of pinch in an incompressible lubricant is to increase the load capacity by a quantity independent of flow rate, the contribution to the load capacity does depend on the flow rate in the case of a compressible lubricant.

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